# Storage of Sugar Cane Bagasse

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# Introduction

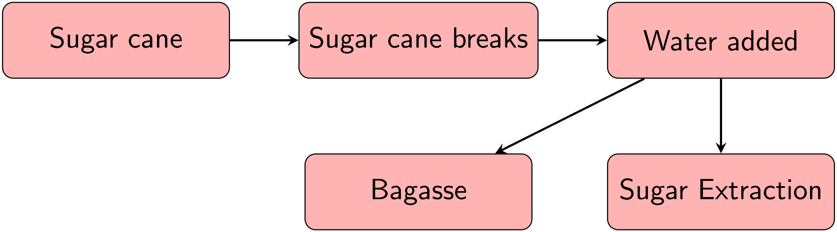



Figure: Moisture levels of 45 - 55%

# Problem Description

- Stockpile as a resource
- Spontaneous combustion
- T. F. Dixon (1988)
- B. F. Gray et al (2002)

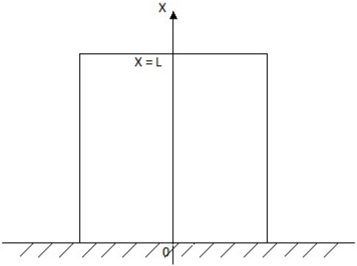



Figure: One Dimensional Model with an insulated bottom

# Desired Outcomes

- Maximum height of the bagasse heap to avoid spontaneous combustion?
- Advantage in adjusting the moisture? (Usable energy per unit area)
- Advantage in pelletizing the bagasse?(Usable energy per unit area)

# 1D-Model formulation: B. F. Gray et. al 2001

**Governing equations**

$$(\rho_b c_b + m_w X c_w)\frac{\partial U}{\partial t} = Q\rho_b Z W \exp(-E/RU) \\ + Q_w \rho_b Z_w X W \exp(-E_w/RU) f(U) \\ + L_v[Z_c Y - Z_e X \exp(-L_v/RU)] + \kappa \nabla^2 U, \quad (1)$$

$$\frac{\partial Y}{\partial t} = Z_e X \exp(-L_v/RU) - Z_c Y + D_Y \nabla^2 Y, \quad (2)$$

$$\frac{\partial X}{\partial t} = -Z_e X \exp(-L_v/RU) + Z_c Y, \quad (3)$$

$$\frac{\partial W}{\partial t} = -F\rho_b Z W \exp(-E/RU) - F\rho_b Z_w X W \exp(-E_w/RU) f(U) \\ + D_w \nabla^2 W. \quad (4)$$

$U$ is temperature, $Y$ is vapour concentration, $X$ is liquid concentration, $W$ is oxygen concentration

# 1D-Model formulation cont'd

**Boundary Conditions**

At the bottom, $x = 0$, we impose the no flow condition (of heat or material)

$$\frac{\partial U}{\partial x} = 0, \qquad \frac{\partial Y}{\partial x} = 0, \qquad \frac{\partial W}{\partial x} = 0, \tag{5}$$

At the top surface, $x = L$,

$$k\frac{\partial U}{\partial x} = h(U - U_a), \; -D_Y\frac{\partial Y}{\partial x} = h_Y(Y - Y_a), \; -D_W\frac{\partial W}{\partial x} = h_W(W - W_a), \tag{6}$$

**Initial Conditions**

$$U(x,0) = U_0(x), \qquad Y(x,0) = Y_0(x), \tag{7}$$
$$X(x,0) = X_0(x), \qquad W(x,0) = W_0(x). \tag{8}$$

# 1D-Model formulation cont'd

**Steadу-state equations**

$$0 = D_Y \frac{\partial^2 Y}{\partial x^2} + Z_e X \exp\left(-\frac{L_v}{RU}\right) - Z_c Y \tag{9}$$

$$0 = -Z_e X \exp\left(-\frac{L_v}{RU}\right) + Z_c Y \tag{10}$$

$$Y_{xx} = 0 \;\Rightarrow Y_s = Y_a,\; X_s = \frac{Z_c Y_a}{Z_e} \exp\left(\frac{L_v}{RU}\right) \tag{11}$$

$$0 = k \frac{\partial^2 U}{\partial x^2} + Q \rho_b Z W \exp\left(-\frac{E}{RU}\right) + Q_w \rho_b Z_w X_s W \exp\left(-\frac{E_w}{RU}\right) f(U) \tag{12}$$

# 1D-Model formulation cont'd

$$0 = D_W \frac{\partial^2 W}{\partial x^2} - F\rho_b Z W \exp\left(-\frac{E}{RU}\right) - F\rho_b Z_w X W \exp\left(-\frac{E_w}{RU}\right) f(U) \tag{13}$$

If bagasse is hot (everywhere above 58C), then

$$\frac{k}{Q}\frac{\partial^2 U}{\partial x^2} + \frac{D_W}{F}\frac{\partial^2 W}{\partial x^2} = 0 \tag{14}$$

Applying boundary conditions at $x = 0$

$$\frac{k}{Q}U + \frac{D_W}{F}W = C_0 \tag{15}$$

$$\Delta Y = Y_a \qquad \Delta W = W_a \qquad \Delta X = \frac{Z_c Y_a}{Z_e}\exp\left(\frac{L_v}{RU_i}\right) \qquad \Delta U = U_i - U_a$$

# Dimensionless form

**Non-dimensional model**

$$\hat{t} = \frac{t}{\Delta t}, \qquad \hat{x} = \frac{x}{L}, \qquad \hat{U} = \frac{U - U_a}{\Delta U}, \qquad \hat{Y} = \frac{Y}{\Delta Y},$$

$$\hat{X} = \frac{X}{\Delta X}, \qquad \hat{W} = \frac{W}{\Delta W}, \tag{16}$$

Diffusion time scale is $$\Delta t = \frac{L^2(\rho_b c_b + m_w c_w \Delta X)}{k} = \frac{L^2}{D_U}, \tag{17}$$

The liquid equation is

$$\frac{1}{Z_e \Delta t} \exp\left(\frac{L_v}{RU_i}\right) \frac{\partial \hat{X}}{\partial \hat{t}} = -\hat{X} \exp\left(\frac{\alpha_{L_v}(\hat{U} - 1)}{U_a + \Delta U \hat{U}}\right) + \hat{Y}, \tag{18}$$

where

$$\alpha_{L_v} = \frac{L_v \Delta U}{RU_i}. \tag{19}$$

# Dimensionless form

Coefficient of LHS is $\mathcal{O}(10^{-5})$, hence

$$\hat{X} = \exp\left(-\frac{\alpha_{L_v}(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right)\hat{Y} \tag{20}$$

Lose terms in heat and vapour equations
Vapour equation

$$\kappa_Y \frac{\partial \hat{Y}}{\partial \hat{t}} = \frac{\partial^2 \hat{Y}}{\partial \hat{x}^2}, \quad \text{where} \quad \kappa_Y = \frac{L^2}{\Delta t D_Y} = \mathcal{O}(10^{-1}). \tag{21}$$

$$(\beta_1 + \beta_2 \hat{X})\frac{\partial \hat{U}}{\partial \hat{t}} = \frac{\partial^2 \hat{U}}{\partial \hat{x}^2} + A_E \hat{W} \exp\left(\frac{\alpha_E(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) + A_{E_w} \hat{X} \hat{W} \exp\left(\frac{\alpha_{E_w}(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) f(\hat{U}), \tag{22}$$

# Dimensionless form

where

$$\beta_1 = \frac{\rho_b c_b L^2}{k\Delta t} \qquad \beta_2 = \frac{m_w c_w \Delta X L^2}{k\Delta t} \tag{23}$$

$$A_E = \frac{Q\rho_b Z\Delta W L^2}{k\Delta U} \exp\left(-\frac{E}{RU_i}\right) \tag{24}$$

$$A_{E_w} = \frac{Q_w\rho_b Z_w\Delta X\Delta W L^2}{k\Delta U} \exp\left(-\frac{E_w}{RU_i}\right), \tag{25}$$

$$\alpha_E = \frac{E\Delta U}{RU_i}, \qquad \alpha_{E_w} = \frac{E_w\Delta U}{RU_i} \tag{26}$$

# Dimensionless form

The oxygen equation becomes

$$\kappa_W \frac{\partial \hat{W}}{\partial \hat{t}} = \frac{\partial^2 \hat{W}}{\partial \hat{x}^2} - B_E \hat{W} \exp\left(\frac{\alpha_E(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) - B_{E_w} \hat{X} \hat{W} \exp\left(\frac{\alpha_{E_w}(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) f(\hat{U}), \quad (27)$$

where

$$\kappa_W = \frac{L^2}{\Delta t D_W} \quad B_E = \frac{F \rho_b Z L^2}{D_W} \exp\left(-\frac{E}{R U_i}\right) \quad (28)$$

$$B_{E_w} = \frac{F \rho_b Z_w \Delta X L^2}{D_w} \exp\left(-\frac{E_w}{R U_i}\right). \quad (29)$$

# Dimensionless form

**Boundary conditions**

$$\text{At} \quad \hat{x} = 0 : \quad \frac{\partial \hat{U}}{\partial \hat{x}} = 0, \qquad \frac{\partial \hat{Y}}{\partial \hat{x}} = 0, \qquad \frac{\partial \hat{W}}{\partial \hat{x}} = 0, \quad \text{at } \hat{x} = 0, \tag{30}$$

$$\text{At} \quad \hat{x} = 1 :$$

$$-\frac{\partial \hat{U}}{\partial \hat{x}} = \gamma \hat{U}, \qquad -\frac{\partial \hat{Y}}{\partial \hat{x}} = \gamma_Y(\hat{Y} - 1), \qquad -\frac{\partial \hat{W}}{\partial \hat{x}} = \gamma_W(\hat{W} - 1), \tag{31}$$

where

$$\gamma = \frac{hL}{k}, \qquad \gamma_Y = \frac{h_Y L}{D_Y}, \qquad \gamma_W = \frac{h_W L}{D_W}. \tag{32}$$

Note $\gamma = \mathcal{O}(10)$, $\gamma_Y = \gamma_W = \mathcal{O}(10^5)$ so we may simplify the boundary conditions $\hat{Y} = \hat{W} = 1$ at $\hat{x} = 1$. The initial conditions are

$$U = U_0, \qquad Y = Y_0, \qquad W = W_0, \quad \text{at } t = 0 \tag{33}$$

# Discussion

**Simplest model**

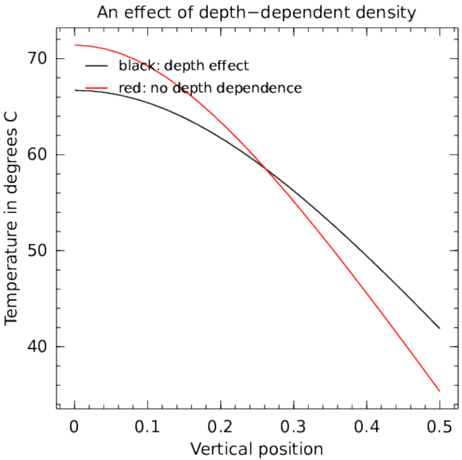
Steady-state temperature

$$0 = \frac{\partial^2 \hat{U}}{\partial \hat{x}^2} + A \exp\left(\frac{\alpha(\hat{U} - 1)}{U_a + \Delta U \hat{U}}\right) \tag{34}$$

This is standard form, but leads to very small piles

# Discussion

What happens when the density is not assumed constant?



# Discussion

**Pseudo steady-state**

$\kappa_W, \kappa_Y$ small

$$\hat{X} = \exp\left(-\frac{\alpha_{L_v}(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right)\hat{Y} \qquad \hat{Y} = 1 \tag{35}$$

$$(\beta_1 + \beta_2 \hat{X})\frac{\partial \hat{U}}{\partial \hat{t}} = \frac{\partial^2 \hat{U}}{\partial \hat{x}^2} + A_E \hat{W} \exp\left(\frac{\alpha_E(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) + A_{E_w} \hat{X} \hat{W} \exp\left(\frac{\alpha_{E_w}(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) f(\hat{U}), \tag{36}$$

$$0 = \frac{\partial^2 \hat{W}}{\partial \hat{x}^2} - B_E \hat{W} \exp\left(\frac{\alpha_E(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) - B_{E_w} \hat{X} \hat{W} \exp\left(\frac{\alpha_{E_w}(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) f(\hat{U}), \tag{37}$$

# Discussion

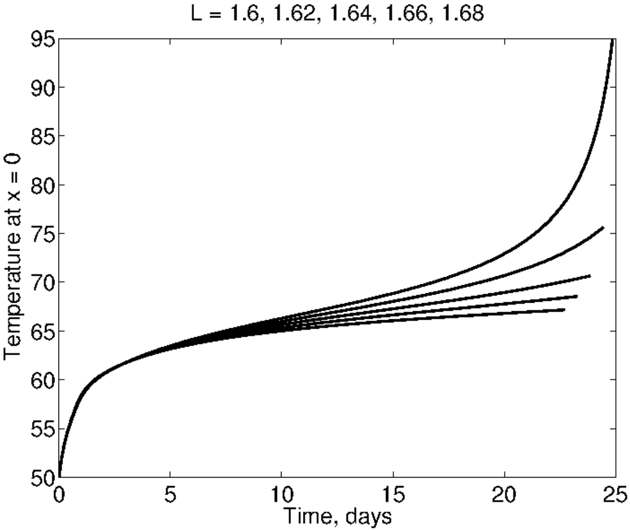
**Almost full problem**

$$\hat{X} = \exp\left(-\frac{\alpha_{L_v}(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right)\hat{Y} \tag{38}$$

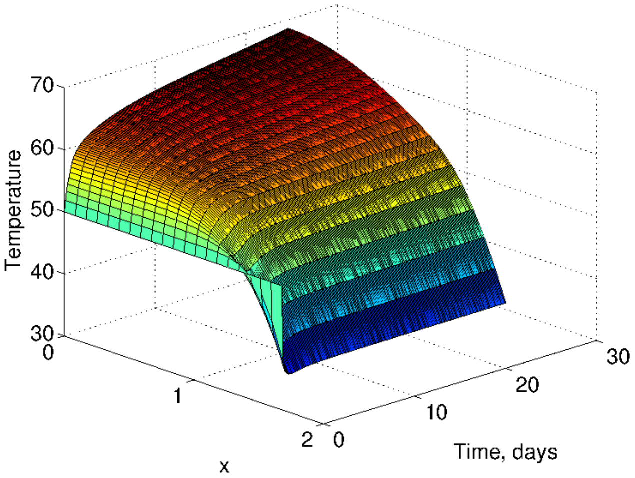
$$\kappa_Y \frac{\partial \hat{Y}}{\partial \hat{t}} = \frac{\partial^2 \hat{Y}}{\partial \hat{x}^2}, \tag{39}$$

$$(\beta_1 + \beta_2 \hat{X})\frac{\partial \hat{U}}{\partial \hat{t}} = \frac{\partial^2 \hat{U}}{\partial \hat{x}^2} + A_E \hat{W} \exp\left(\frac{\alpha_E(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) + A_{E_w}\hat{X}\hat{W}\exp\left(\frac{\alpha_{E_w}(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) f(\hat{U}), \tag{40}$$

$$\kappa_W \frac{\partial \hat{W}}{\partial \hat{t}} = \frac{\partial^2 \hat{W}}{\partial \hat{x}^2} - B_E \hat{W} \exp\left(\frac{\alpha_E(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) - B_{E_w}\hat{X}\hat{W}\exp\left(\frac{\alpha_{E_w}(\hat{U}-1)}{U_a + \Delta U \hat{U}}\right) f(\hat{U}), \tag{41}$$

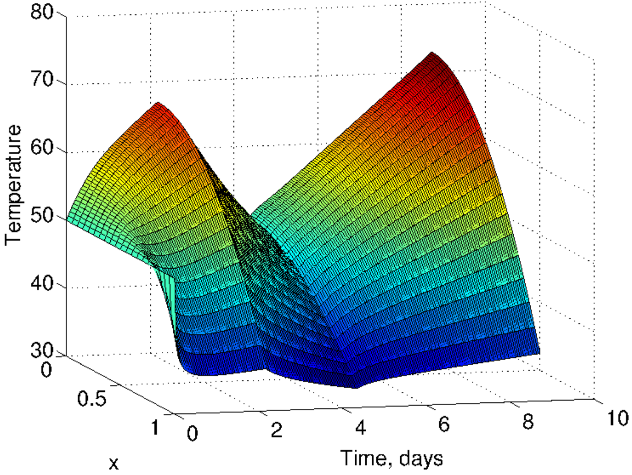



Note, insulated bottom and 100% humidity. Pile height increases with lower humidity




Typical evolution of temperature

Often appears piles can be very large without ignition but ...




What if it rains?

# Ignition model — words

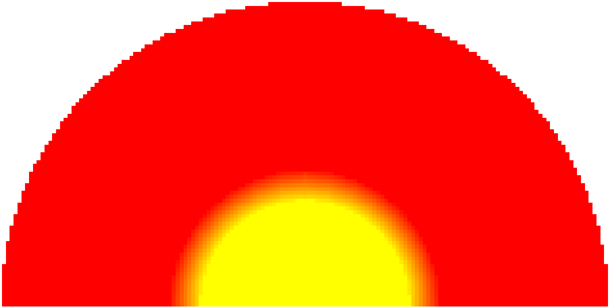
Puzzle: why do apparently stable heaps ignite after getting soaked?

— wet reaction is fast, but turns off for temperatures above 58 °C,
— dry reaction is slower
So, near centre the bagasse dries out and starts to heat above the 58 °C limit. We imagine two steady states:

**inner:** hot and dry, insulated inner end, at 58 °C at interface
**outer:** warm and wet, 58 °C at interface, cooling condition at surface

Stefan problem with moving boundary

# Ignition model — diagram



Please use conformal mapping to imagine this as a square with a hot yellow and a warm red band . . .

We have a good handle on equations for the steady states, but haven't got a formulation for the velocity of the moving interface

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- Under normal conditions pile does not burn, but adding water can then cause ignition

# Conclusion

- We have a model for temperature evolution in bagasse piles - can be made simpler
- Steady-state models should be sufficient - to provide bifurcation diagram
- For any ambient conditions we can cause ignition, by making the pile sufficiently large
- Under normal conditions pile does not burn, but adding water can then cause ignition
- We have looked at a worst case scenario - insulated bottom, no heat loss at sides. Model can be improved.
- Future work will constitute consideration of a more realistic boundary condition at the bottom, 2D model with heat loss at the sides; compare full system to simplified models.

# References


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